

Stress and strain tensors pdf

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The maximum shear stress at a point in a continuum body is determined by maximizing τ subject to the condition that $n_1 = n_1 + n_2 + n_3 = 1$. This is a constrained maximization problem, which can be solved using the Lagrangian multiplier technique to convert the problem into an unconstrained optimization problem. Thus, the stationary values (maximum and minimum values) of τ occur where the gradient of τ is parallel to the gradient of F . The Lagrangian function for this problem can be written as $F(n_1, n_2, n_3, \lambda) = \tau + \lambda(g(n_1, n_2, n_3) - 1)$, where λ is the Lagrangian multiplier (which is different from the λ used to denote eigenvalues). The extreme values of these functions are $\partial F / \partial n_1 = 0$, $\partial F / \partial n_2 = 0$, $\partial F / \partial n_3 = 0$, and $\partial F / \partial \lambda = 0$. The partial derivatives of F with respect to n_1, n_2, n_3 and λ are given by:

$$\frac{\partial F}{\partial n_1} = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_{12}^2 - \sigma_{13}^2 - \sigma_{23}^2 + \lambda(n_1 - 1)$$

$$\frac{\partial F}{\partial n_2} = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_{12}^2 - \sigma_{13}^2 - \sigma_{23}^2 + \lambda(n_2 - 1)$$

$$\frac{\partial F}{\partial n_3} = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_{12}^2 - \sigma_{13}^2 - \sigma_{23}^2 + \lambda(n_3 - 1)$$

$$\frac{\partial F}{\partial \lambda} = g(n_1, n_2, n_3) - 1$$

These three equations together with the condition $n_1 + n_2 + n_3 = 1$ may be solved for λ , n_1 , n_2 , and n_3 . By multiplying the first three equations by n_1, n_2 , and n_3 , respectively, and knowing that $\sigma = \sigma_1 n_1 + \sigma_2 n_2 + \sigma_3 n_3$, we obtain $n_1^2 + n_2^2 + n_3^2 - 2\sigma_{12}n_1 - 2\sigma_{13}n_2 - 2\sigma_{23}n_3 = 0$. Adding these three equations we get $n_1^2 + n_2^2 + n_3^2 - 2\sigma_{12}n_1 - 2\sigma_{13}n_2 - 2\sigma_{23}n_3 = 0$. A first approach to solve these last three equations is to consider the trivial solution $n_1 = 0$, $n_2 = 0$, $n_3 = 0$. However, this option does not fulfill the constraint $n_1 + n_2 + n_3 = 1$. Considering the solution where $n_1 = n_2 = 0$ and $n_3 \neq 0$, it is determined from the condition $n_1 + n_2 + n_3 = 1$ that $n_3 = \pm 1$. It is seen that $\tau = 0$ for $n_1 = n_2 = 0$, $n_3 = \pm 1$. The other two possible values for τ can be obtained similarly by assuming $n_1 = n_2 = 0$ and $n_3 = \pm 1$. Thus, one set of solutions for these four equations is: $n_1 = 0, n_2 = 0, n_3 = \pm 1$, $\sigma_1 = 0, \sigma_2 = 0, \sigma_3 = \pm 1$, $\tau = \pm 1$.

A second set of solutions is obtained by assuming $n_1 = 0, n_2 \neq 0, n_3 \neq 0$. Then solving for n_3 we have $n_3 = \pm 1$. Then solving for n_2 we have $n_2 = \pm 1$. Then solving for n_1 we have $n_1 = \pm 1$. The other two possible values for τ can be obtained similarly by assuming $n_1 = 0, n_2 \neq 0, n_3 \neq 0$. Therefore, the second set of solutions for τ is $n_1 = 0, n_2 = \pm 1, n_3 = \pm 1$, $\sigma_1 = 0, \sigma_2 = 0, \sigma_3 = \pm 1$, $\tau = \pm 1$.

The maximum shear stress is expressed by $\tau_{max} = \sqrt{(\sigma_1 - \sigma_3)^2/2}$ and it can be stated as being equal to one-half the difference between the largest and smallest principal stresses, acting on the plane that bisects the angle between the directions of the largest and smallest principal stresses. Stress deviator tensor s_{ij} can be expressed as the sum of two other stress tensors: a mean hydrostatic stress tensor or mean normal stress tensor, \bar{s}_{ij} , which tends to change the volume of the stressed body; and a deviatoric component called the stress deviator tensor, $s_{ij} - \bar{s}_{ij}$, where $\bar{s}_{ij} = \sigma_{11} + \sigma_{22} + \sigma_{33}/3$. Pressure (p) is generally defined as negative one-third the trace of the stress tensor minus any stress the divergence of the velocity contributes with, i.e. $p = \lambda \nabla \cdot u - \pi$, where λ is the proportionality constant, $\nabla \cdot u$ is the divergence operator, and π is the Cartesian coordinate of u . The deviatoric stress tensor can be obtained by subtracting the hydrostatic stress tensor from the Cauchy stress tensor: $s_{ij} = s_{ij} - \bar{s}_{ij}$.

Octahedral stress planes Considering the principal directions as the coordinate axes, a plane whose normal vector makes equal angles with each of the principal axes (i.e. having direction cosines equal to $1/\sqrt{3}$) is called an octahedral plane. There are a total of eight octahedral planes (Figure 6). The normal and shear components of the stress tensor on these planes are called octahedral normal stress σ_{oct} and octahedral shear stress τ_{oct} respectively. Octahedral plane passing through the origin is known as the π -plane (π not to be confused with mean stress denoted by $\bar{\pi}$ in above section). On the π -plane, $s_{ij} = 1/3 I$. Knowing that the stress tensor of point O (Figure 6) in the principal axes is $s_{ij} = [1 0 0 0 0 0 0 0 0]$, the stress vector on an octahedral plane is then given by: $\sigma_{oct} = \sigma_{11} + \sigma_{22} + \sigma_{33}$. The normal component of the stress vector at point O is $\sigma_{oct} = T_i(n_i)$, where $T_i(n_i) = \sqrt{(\sigma_{11} - \sigma_{22})^2/2 + (\sigma_{22} - \sigma_{33})^2/2 + (\sigma_{33} - \sigma_{11})^2/2}$. The shear component of the stress vector at point O is $\tau_{oct} = T_i(n_i) - \sigma_{oct}$.

Invariants of the stress deviator tensor As it is a second order tensor, the stress deviator tensor also has a set of invariants, which can be obtained using the same procedure used to calculate the invariants of the stress tensor. It can be shown that the principal directions of the stress deviator tensor s_{ij} are the same as the principal directions of the stress tensor σ_{ij} . Thus, the characteristic equation is $|s_{ij} - \lambda| = 0$, where $\lambda_1 = \sigma_{11} - \sigma_{22} - \sigma_{33}/3$, $\lambda_2 = \sigma_{22} - \sigma_{33}/3$, and $\lambda_3 = \sigma_{33} - \sigma_{11}/3$. Their values are the same (invariant) regardless of the orientation of the coordinate system chosen. These deviatoric stress invariants can be expressed as a function of the components of s_{ij} or its principal values s_1, s_2, s_3 , or alternatively, as a function of σ_{ij} or its principal values $\sigma_1, \sigma_2, \sigma_3$. Thus, $s_1 = \sigma_{11} - \sigma_{22} - \sigma_{33}/3$, $s_2 = \sigma_{22} - \sigma_{33}/3$, and $s_3 = \sigma_{33} - \sigma_{11}/3$. Because $s_{ij} = s_{ij} - \bar{s}_{ij}$, the equivalent stress or von Mises stress is commonly used in solid mechanics. The equivalent stress is defined as $\sigma_{eqM} = \sqrt{s_1^2 + s_2^2 + s_3^2}$.

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